Candidacy Exam
Department of Physics
February 5, 2011
Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales		$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$
1 large calorie (as in nutrition)		4.184 kJ
1 inch		2.54 cm

I–1. A particle of mass m moves in a central force field that has constant magnitude F_0, but always points toward the origin.

(a) Find the angular velocity ω_ϕ required for the particle to move in a circular orbit of radius r_0.

(b) Find the frequency of small oscillations about the orbit.

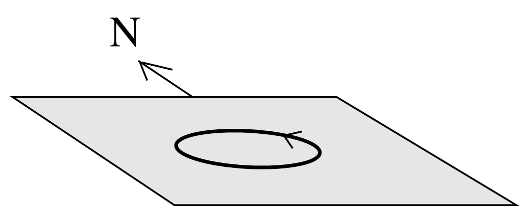
(Both answers should be in terms of F_0 , m and r_0.)

I–2. A circular coil of 18 cm diameter containing 11 loops of conducting wire lies flat on the ground. The Earth's magnetic field at this location has magnitude 5.50×10^{-5} Tesla pointing into the Earth at a vertical angle 56.0° below a line pointing due north.

Suppose a 7.70 A counterclockwise current passes through the coil:

(a) Determine the torque on the coil.

(b) Determine which edge of the coil, north, south, east, or west, lifts up from the ground. (Assume the coil is light enough.)



I–3. The Hamiltonian of a spin 1 system if

$$H = AS_z^2 + B(S_x^2 - S_y^2), \tag{I–1}$$

where S_x, S_y, and S_z are spin operators.

(a) What is a suitable matrix representation of the spin operators, given their commutation relations.

(b) Find the eigenvalues and the normalized eigenstates of H.

I–4. In a temperature range around temperature T the tension force of a stretched plastic rod is related to its length by

$$F = aT^2(L - L_0), \tag{I–2}$$

where a and L_0 are positive constants. When $L = L_0$ the heat capacity measured at constant length is given by

$$C_L(T, L_0) = bT, \tag{I–3}$$

where b is a constant.

(a) Write the fundamental thermodynamic relation for this system, expressing dS in terms of dE and dL.

(b) The entropy of the rod is a function of temperature and length. Find

$$\left(\frac{\partial S}{\partial L}\right)_T . \tag{I–4}$$

(c) Knowing $S(T_0, L_0)$, where T_0 and L_0 are some constant temperature and length, find the entropy $S(T, L)$ for any other temperature and length.

(d) Find the heat capacity $C_L(T, L)$ of the rod when its length is L instead of L_0.

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II–1. A helium balloon and its gondola, mass M, are in the air and stationary with respect to the ground. There is no wind.

A passenger, mass m, climbs out and slides down a rope with constant speed v, measured with respect to the balloon.

(a) With what speed and direction (relative to the Earth) does the balloon move?

(b) What happens if the passenger stops sliding down the rope?

II–2. A particle of charge e and mass m is initially at rest in a magnetic field $\mathbf{B} = \hat{\mathbf{z}}B$. At $t = 0$ an electric field $\hat{\mathbf{x}}E$ is suddenly switched on, and left on. Describe the subsequent motion of the particle using classical physics.

II–3. Of the energy input to a 60 W incandescent light bulb, about 2% is radiated as visible light (wavelengths 400 nm–700 nm).

(a) How many photons are emitted per second?

A night-adapted eye detects around 100 photons per second.

(b) Given that the pupil of the eye is 0.6 cm in diameter in darkness, very roughly determine from what distance is it possible for the naked eye to detect photons from a 60 W light bulb?

II–4. A certain system (which is actually a model of a black hole) is a spherical object of surface area A and temperature T. Its internal energy depends on its area as $U = a\sqrt{A}$. The object also obeys the thermodynamical relation $U(T) = bT^{-1}$. Here a and b are positive constants. (Thus A is a function of T.)

(a) Show that the system is unstable by computing its specific heat.

(b) Use Stefan's law to determine the rate of energy loss if the sphere is placed in a large reservoir much colder than T. Find A as a function of time, given an initial value $A = A_0$. Show that the internal energy vanishes after a finite amount of time.

(Treat the radiation as if the system were a perfect black body.)