

Candidacy Exam  
 Department of Physics  
 August 20, 2005

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

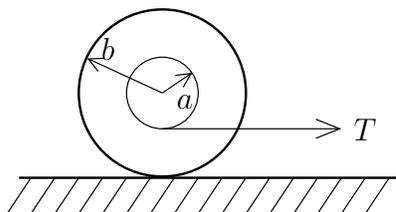
Fundamental constants:

Avogadro's number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's constant	$k_B$	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Electron charge magnitude	$e$	$1.602 \times 10^{-19} \text{ C}$
Gas constant	$R$	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Planck's constant	$h$	$6.626 \times 10^{-34} \text{ J s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34} \text{ J s}$
Speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m s}^{-1}$
Permittivity constant	$\epsilon_0$	$8.854 \times 10^{-12} \text{ F m}^{-1}$
Permeability constant	$\mu_0$	$1.257 \times 10^{-6} \text{ N A}^{-2}$
Gravitational constant	$G$	$6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^5 \text{ N m}^{-2}$
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Electron rest mass	$m_e$	$9.109 \times 10^{-31} \text{ kg} = 0.5110 \text{ MeV } c^{-2}$
Proton rest mass	$m_p$	$1.673 \times 10^{-27} \text{ kg} = 938.3 \text{ MeV } c^{-2}$
Origin of temperature scales	$0^\circ\text{C} = 273 \text{ K}$	

- I-1. A reel consists of a cylindrical hub of radius  $a$  and two circular end pieces of radius  $b$ . The mass of the complete reel is  $m$  and its moment of inertia about its long axis is  $I$ . The reel rests on horizontal table. The end of a light string is attached to the hub and wrapped around it, and a tension  $T$  in a horizontal direction is applied to the free end of the string, as shown in the figure. The coefficient of friction is large enough that the reel rolls on the table.

Determine:

- (a) the frictional force exerted by the table,
- (b) the direction in which the reel begins to move.



- I-2. Two argon atoms are located along the  $x$ -axis at  $x = \pm a$ . Each has an isotropic polarizability  $\alpha$ , such that the atom develops a dipole moment  $\mathbf{p} = \alpha \mathbf{E}_l$  when subject to a local electric field  $\mathbf{E}_l$ . Obtain the total dipole moment of the two-atom system under each of the following conditions:

- (a) When a constant electric field of size  $E_0$  is imposed along the  $x$ -axis.  
( $\mathbf{E} = \hat{\mathbf{x}}E_0$ )
- (b) When a constant electric field of size  $E_0$  is imposed along the  $y$ -axis.  
( $\mathbf{E} = \hat{\mathbf{y}}E_0$ )

- I-3. According to simple kinetic theory, the thermal conductivity of a gas is given by the expression

$$K = \frac{1}{3} C_V \lambda \langle v \rangle, \quad (\text{I-1})$$

where  $C_V$  is the heat capacity of the gas at constant volume, per unit volume,  $\lambda$  is the mean distance between collisions, and  $\langle v \rangle$  is the mean speed of the molecules. Approximate all molecules as spherical, and until part (d) ignore their internal vibrations, and treat the remaining degrees of freedom as classical.

- (a) Find  $\lambda$  in terms of the number density of the gas and the radius of the molecules.
- (b) How do the quantities on the right-hand-side of Eq. (I-1) depend on temperature  $T$ ? Deduce the temperature dependence of  $K$ .
- (c) Regard methane gas as a collection of spherical molecules 1.7 times the radius of argon atoms. The atomic weight of argon is 40, and the molecular weight of methane is 16. Estimate the ratio of the thermal conductivity of methane to argon gas, given that the gases have the same number density. [Hint: The molar heat capacity at constant volume,  $c_V$ , is  $Rf/2$  for a gas with  $f$  degrees of freedom.]
- (d) Would we expect this ratio to increase or decrease if the methane molecules were vibrationally excited by collisions.

I-4. Explain the significance of the photoelectric effect for the discovery of quantum mechanics.

In a photoelectric experiment monochromatic light of wavelength  $\lambda$  falls on a potassium surface. It is found that the stopping potential is 1.91 V for  $\lambda = 3000 \text{ \AA}$ , and 0.88 V for  $\lambda = 4000 \text{ \AA}$ . From these data calculate

- (a) A value for Planck's constant given the values for the size of the charge of the electron  $e$  and for the speed of light  $c$  that are given in the table at the front of this exam.
- (b) The work function  $W$  for potassium.
- (c) The threshold frequency  $\nu_t$  for potassium.

Candidacy Exam  
 Department of Physics  
 August 20, 2005

Part II

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

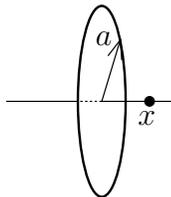
Fundamental constants:

Avogadro's number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's constant	$k_B$	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Electron charge magnitude	$e$	$1.602 \times 10^{-19} \text{ C}$
Gas constant	$R$	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Planck's constant	$h$	$6.626 \times 10^{-34} \text{ J s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34} \text{ J s}$
Speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m s}^{-1}$
Permittivity constant	$\epsilon_0$	$8.854 \times 10^{-12} \text{ F m}^{-1}$
Permeability constant	$\mu_0$	$1.257 \times 10^{-6} \text{ N A}^{-2}$
Gravitational constant	$G$	$6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^5 \text{ N m}^{-2}$
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Electron rest mass	$m_e$	$9.109 \times 10^{-31} \text{ kg} = 0.5110 \text{ MeV } c^{-2}$
Proton rest mass	$m_p$	$1.673 \times 10^{-27} \text{ kg} = 938.3 \text{ MeV } c^{-2}$
Origin of temperature scales	$0^\circ\text{C} = 273 \text{ K}$	

II-1. The muon is a particle with mass  $m_\mu = 106 \text{ MeV}/c^2$  and mean lifetime  $\tau_\mu = 2.20 \mu\text{s}$ . A beam is made of muons of energy  $100 \text{ GeV}$ .

- (a) By what fraction does the speed of the muons deviate from the speed of light  $c$ ?
- (b) What is the distance the beam travels before a fraction  $1/e$  of the muons have decayed?

II-2. An electron is placed at the center of a uniform thin ring of charge  $q$  and radius  $a$ . The mass of the electron is  $m$ . It is then displaced a small distance  $x$  along the axis of the ring ( $x \ll a$ ). Derive a formula for the frequency of small oscillations.



II-3. An air bubble of  $20 \text{ cm}^3$  volume is at the bottom of a lake  $40 \text{ m}$  deep where the temperature is  $4.0^\circ\text{C}$ . The bubble rises to the surface, which is at a temperature of  $20^\circ\text{C}$ . Take the temperature of the bubble to be the same as that of the surrounding water and find its volume just before it reaches the surface.

II-4. Consider a particle of mass  $m$  constrained to move in one dimension, along the  $x$ -axis. The particle experiences a harmonic oscillator potential  $V(x) = \frac{1}{2}kx^2$ . A natural angular frequency for the oscillator is defined by  $\omega = \sqrt{k/m}$ .

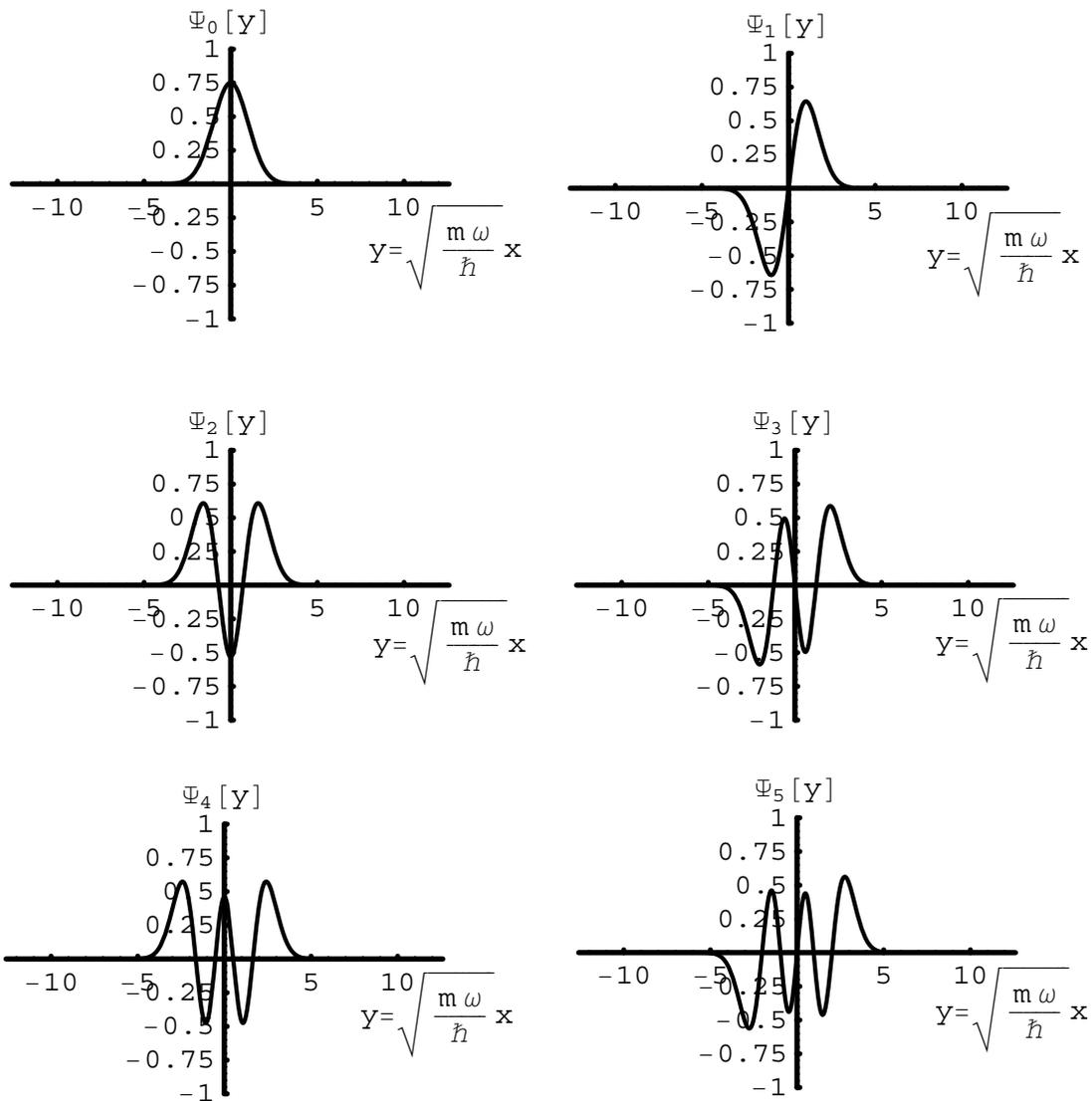
The first 6 normalized wave functions (energy eigenfunctions) are shown in Fig. II-1 on page II-3. Note that the wave functions are expressed as functions of the dimensionless variable  $y \equiv x\sqrt{m\omega/\hbar}$  and are normalized so that  $\int_{-\infty}^{\infty} \psi_n^*(y)\psi_n(y) dy = 1$ .

- (a) Write down the energies associated with each of the first 6 wave functions.
- (b) Now assume that the potential  $V(x)$  is modified to be infinite for  $x < 0$ ,

$$V_1(x) = \begin{cases} \frac{1}{2}kx^2 & \text{for } x \geq 0, \\ \infty & \text{for } x < 0. \end{cases} \quad (\text{II-1})$$

Determine the three lowest energy eigenvalues for the system when the potential is  $V_1(x)$ .

- (c) Assume the system initially has potential  $V_1(x)$  and is in the ground state for that potential. The potential then suddenly changes to  $V(x)$ . Calculate the probability that the system is in the new ground state after the sudden change in potential. *Note: Take careful note of the details of the normalization condition that is applied to the wave functions.*



$$\Psi_0(y) = e^{-y^2/2} \frac{1}{\pi^{1/4}},$$

$$\Psi_2(y) = e^{-y^2/2} \frac{-1 + 2y^2}{\sqrt{2} \pi^{1/4}},$$

$$\Psi_4(y) = e^{-y^2/2} \frac{3 - 12y^2 + 4y^4}{2\sqrt{6} \pi^{1/4}}$$

$$\Psi_1(y) = e^{-y^2/2} \frac{\sqrt{2} y}{\pi^{1/4}},$$

$$\Psi_3(y) = e^{-y^2/2} \frac{-3y + 2y^3}{\sqrt{3} \pi^{1/4}}$$

$$\Psi_5(y) = e^{-y^2/2} \frac{15y - 20y^3 + 4y^5}{2\sqrt{15} \pi^{1/4}}$$

Figure II-1: Plots of wave functions for Problem II-4.